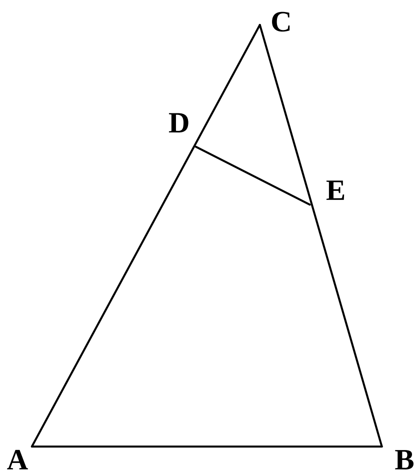
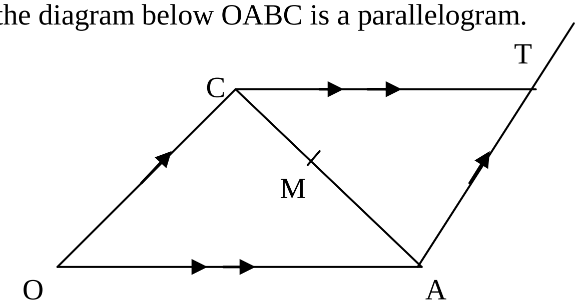
VECTORS

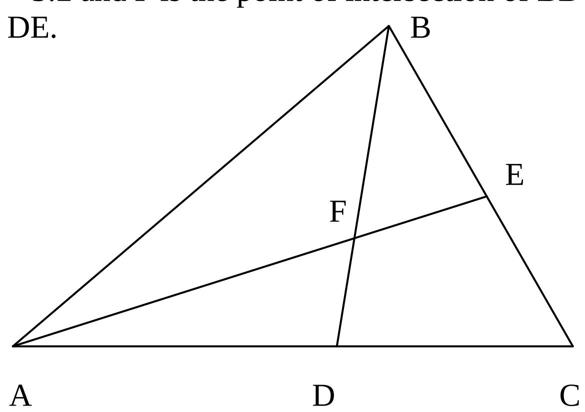
KCSE 1989 – 2012 Form 3 Mathematics		Working Space
1.	**1989 Q11 P2** In the figure below, **AB** = **P**, **AD** = $^3/_5$ **AC** and **CE** = $^2/_3$ **CB**  Express **DE** in terms of **p** and **q**	
2.	**1990 Q21 P1** In a parallelogram ABCD, **AB** = 2**a** and **AD** = **b**. M is the midpoint of AB. AC cut MD at X. i) Express AC in terms of **a** and **b** (2 marks) ii) Given that **AX** = m**AC** and **MX** = n**MD**, where m and n are constants, find m and n. (6 marks)	Working Space

3.	**1990 Q8 P2** In a triangle ABC, D is the midpoint of AB and E is appoint on BC such that BE = $^{2}/_{3}$ BC. If **AD** = **P** and **AC** = **Q**, express **EC** in terms of **p** and **q**. (2 marks)	
4.	**1990 Q10 P2** A point T divides a line AB internally in the ratio 5 : 2. Given that A is (-4, 10) And B is (10, 3) find the coordinates of T. (4 marks)	
5.	**1991 Q6 P1** In the diagram below OABC is a parallelogram.  AB is produced to T such that BT: AB = 1**:**2. M is the midpoint of AC. Given that **OA = a** and **OC = c**. Express MT in term of a and c. (3 marks)	Working Space

6.

1991 Q20 P1

In the figure below E is the midpoint of BC, AD: DC = 3:2 and F is the point of intersection of BD and DE.



i) Given that **AB = b** and **AC = c** express **AE** and **BD** in terms of b and c (3 marks)

ii) Given further that **BF** = t**BD** and **AF** = s**AE**

find the values of s and t. (5 marks)

7.

1992 Q11 P1

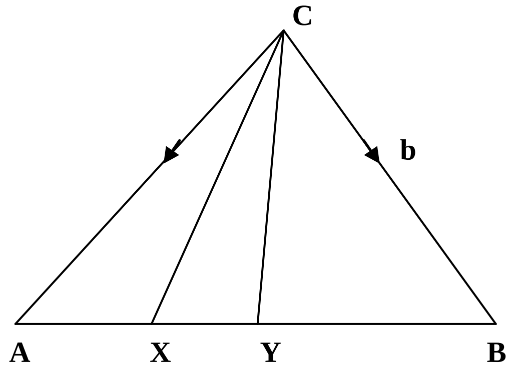
Three points A, B and P are in straight line such that **AP** = t**AB**. Given that the coordinates of A, B and P are (3,4) (8,7) and (x,y) respectively, express x and y in term s of t. (3marks)

Working Space

8.	**1992 Q24 P1** OABC is a trapezium such that the coordinates of O,A,B and C ARE (0,0),(2,-1), (4, 3) and (0, y). a) Find the value of y (2 marks) b) M is a midpoint of AB and N is a midpoint of OM. Show that A, N and C are collinear. (6 marks)	
9.	**1992 Q7 P2** The vectors **p, q** and y are expressed in terms of the vectors t and s as follow: **p** = 3**t** + 2s **q** = 5**t** – s **y** = h**t** + (h – k)s where h and k are constants. Given that y = 2**p** – 3**q**, find the values of and k. (4marks)	
10	**1993 Q21 P1** OABC is a trapezium in which **OA = a**, **OC = c** and **CB = 3a**. CB is produced to such that CB : BD = 3: 1. E is a point on AB such that **AB = 2AE**. Show that O, E and d are collinear. (8 marks)	

11. **1993 Q16 P1**

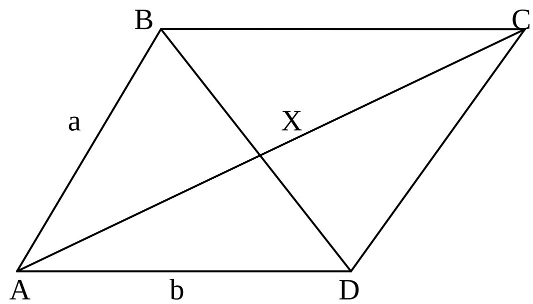
In the figure below **CA** = b **CB** = a, **AX** = **XY** and **AY** = **YB**.



Express CX in terms of a and b (3 marks)

12. **1994 Q24 P1**

In the figure below AB = a, AD = b, AX : XC = 2:3 and XB = 4:5



a) Express

i) AC

ii) **DC** in terms of **a** and **b** in the simplest form. (6 marks)

b) If **DC** = *n***a** + *m***b**, find the values of *n* and *m* (2 marks)

13. **1994Q12P2**

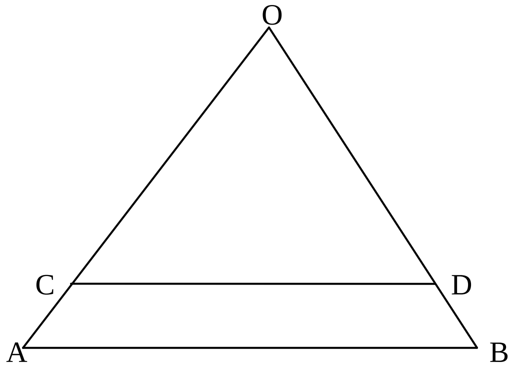
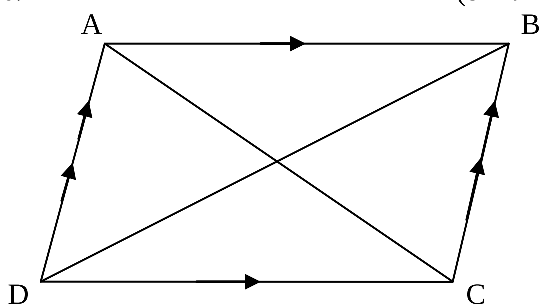
Find the position vector of point R which divides line MN internally in the ratio 2 : 3.

Take the position vectors of M and N to be

M = $\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$ and **N** = $\begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$

(3 marks)

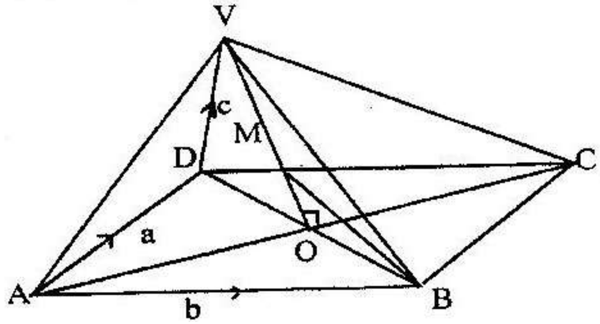
Working Space

14	**1994 Q10 P2** In the figure below OC = 3 CA and OD = 3DB. By taking OA = a, OB = b, show that CD // AB. (3 marks)  	
15.	**1994 Q15 P2** In the figure below ABCD is a parallelogram. AOC and BOD are diagonals of the parallelogram. Show that the diagonals of the parallelogram bisect each other. Give reasons. (3 marks)  	Working Space

16

1995 Q 18 P1

The figure below is a right pyramid with a rectangular base ABCD and VO as the height. The vectors **AD**= a, **AB** = b and **DV** = c



a) Express (i) **AV** in terms of **a** and **c** (1 mark)

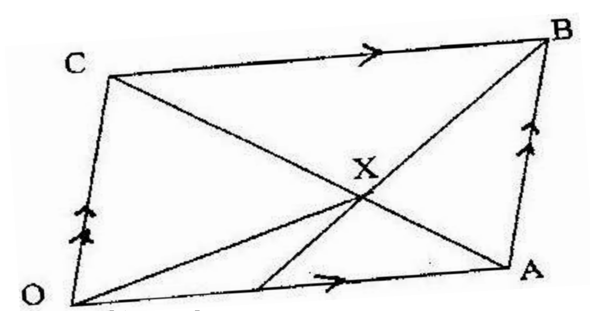
(ii) **BV** in terms of **a**, **b** and **c** (2 marks)

(b) M is point on **OV** such that **OM: MV** = 3:4, Express **BM** in terms of **a, b** and **c** . Simplify your answer as far as possible (5 marks)

17

1996 Q 22 P1

a) In the diagram below OABC is a parallelogram, **OA** = a and **AB** = b. N is a point on **OA** such that **ON: NA** = 1: 2



(b) Find

(i) **AC** in terms of **a** and **b**

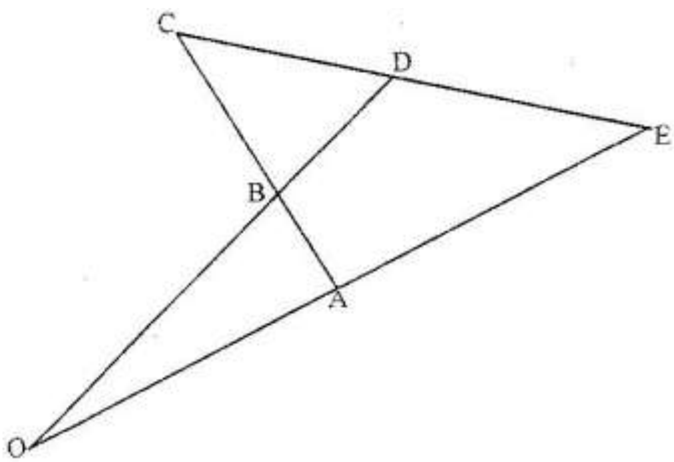
(ii) **BN** in terms of **a** and **b**

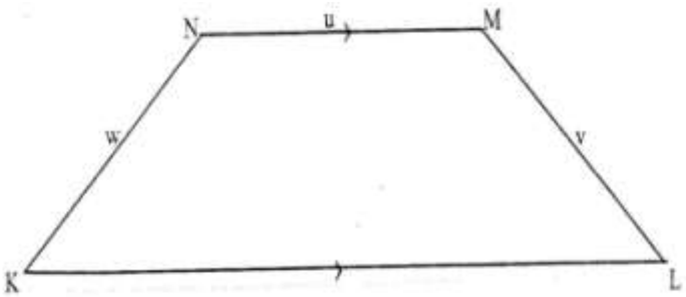
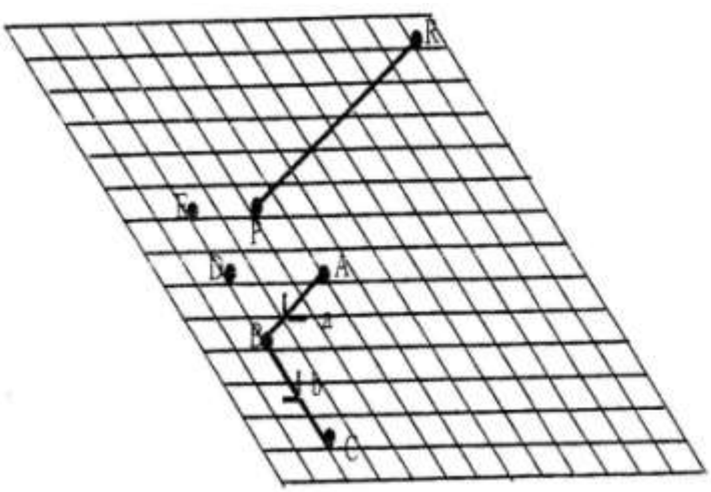
(c) The lines AC and BN intersect at X, **AX** = h**AC** and **BX** = k**BN**

(i) By expressing **OX** in two ways, find the values of h and k

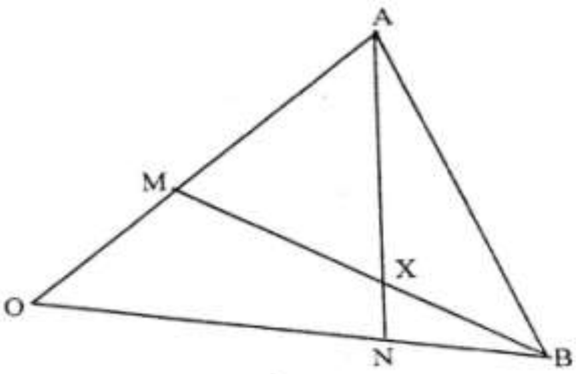
(ii) Express **OX** in terms of **a** and **b** (1 mark)

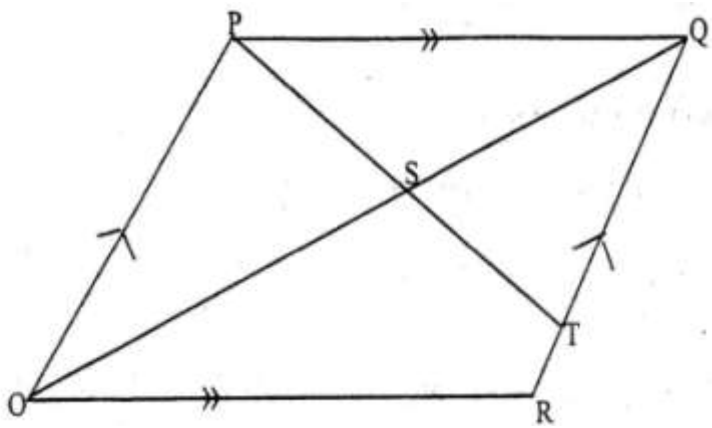
Working Space

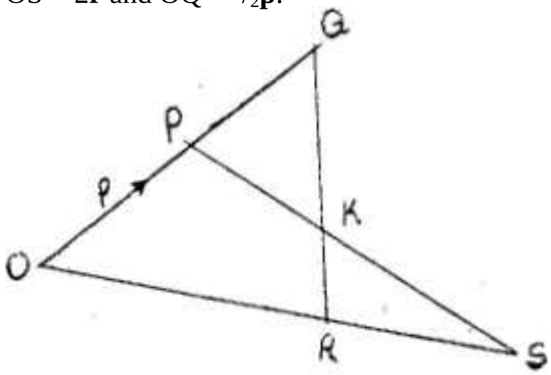
18	**1997 Q 11 P2** ABC is a triangle and P is on AB such that P divides AB internally in the ratio 4:3. Q is a point on AC such that PQ is parallel to BC. If AC = 14 cm (i) State the ratio AQ:QC (ii) Calculate the length of QC	
19.	**1997 Q 22 P1** In the figure below **OA** = a, **OB** = b, **AB** = **BC** and **OB: BD** = 3:1  (a) Determine **(i) AB** (ii) **CD**, in terms of a and b (b) If CD : DE = 1:k and OA:AE = 1: m determine (i) DE in terms of **a, b** and k	Working Space

20	**1998 Q 9 P2** In the figure, KLMN is a trapezium in which KL is parallel to NM and **KL** = 3 **NM**  Given that **KN** = w, **NM** = u and **ML** = v. Show that 2 u = v= w	
21	**1998 Q 22 P1** The figure below shows a grid of equally spaced parallel lines $\overrightarrow{AB}$ = **a** and $\overrightarrow{BC}$ = **b**  	Working Space

	(a) Express (i) $\overrightarrow{AC}$ in terms of a and b (ii) $\overrightarrow{AD}$ in terms of a and b. (b) Using triangle BEP, express $\overrightarrow{BP}$ in terms of a and b (c) PR produced meets BA produced at X and $\overrightarrow{PR} = {}^{1}/_{9}\,\mathbf{b} - {}^{8}/_{3}\mathbf{a}$ By writing $\overrightarrow{PX}$ as $k\overrightarrow{PR}$ and $\overrightarrow{BX}$ as $h\overrightarrow{BA}$ and using the triangle BPX determine the ratio PR: RX	
22	**1999 Q 14 P2** The points P. Q and R lie on a straight line. The position vectors of P and R are 2i + 2j + 13k and 5i – 3j + 4k respectively. Q divides PR Internally in the ratio 2:1. Find the (a) Position vector of Q. (b) Distance of Q from the origin	
23	**1999 Q 21 P1** In triangle OAB, **OA** = **a, OB** = **b** and P lies on AB such that AP: BP = 3:5 (a) Find the terms of **a** and **b** the vectors (i) **AB** (ii) **AP** (iii) **BP** (iv) **OP** (b) Point Q is on OP such AQ = $\frac{-5}{8}$ **a** + $\frac{9}{40}$ _**b.** Find the ratio OQ: QP	
		Working Space

24	**2000 Q 21 P1** The figure below shows triangle OAB in which M divides OA in the ratio 2: 3 and N divides OB in the ratio 4:1 AN and BM intersect at X.  (a) Given that OA = **a** and OB = **b**, express in terms of a and b: (i) **AN** (ii) **BM** (b)If **AX** = s **AN** and **BX** = t**BM**, where s and t are constants, write two expressions for **OX** in terms of **a,b** s and t. Find the value of s Hence write **OX** in terms of a and b	
25	**2001 Q 16 P1** The position vectors for points P and Q are 4 **i** + 3 **j** + 2**k** and 3**i** – 6**j** + 6**k** respectively. Express vector **PQ** in terms of unit vectors I, j and k. Hence find the length of **PQ**, leaving your answer in simplified form.	Working Space

26	**2001 Q 19 P1** The figure below shows a parallelogram OPQR with O as the origin, **OP** = **p** and **OR = r**, Point T divides RQ in the ratio 1:4 and PT Meets OQ at S.  (a) Express in terms of p and r the vectors (i) **OQ** (ii) **OT** (b) Vector **OS** can be expressed in two ways: m**OQ** or **OT** + n **TP**, Where m and n are constants express **OS** in terms of (i) m, **p** and **r** (ii) n, **p** and **r** Hence find the: (iii) Value on m (iv) Ratio OS: SQ	
27	**2002 Q 10 P2** The coordinates of points O,P,Q and R are (0,0) , (3,4) , (11,6) and (8,2) respectively. A point T is such that vectors **OT,QP** and **QR** satisfy the vector equation. **OT** = **QP** + ½ **QR** .Find the coordinates of T.	
28	**2002 Q 4 P1** The position vectors of points X and Y are x=2**i** + **j** – 3**k** and y =3i + 2j -2k respectively. Find **XY**	Working Space

29	**2003 Q 6 P1** Given that x = 2**i** +**j** – 2**k**,y = **-** 3**i** +4**j** –**k** and z = **-** 5**i** +3**j** +2**k** and that p = 3**x** – **y** +2**z**. Find the magnitude of vector p to 3 significant figure (4mks)	
30	**2003 Q 21 P1** In the figure below, vector OP = **p** and OR =**r**. Vector OS = 2**r** and OQ = $^3/_2$**p**.  a) Express in terms of p and r (i) **QR** and (ii) **PS** b) The lines QR and PS intersect at K such that **QK** = m **QR** and **PK** = n **PS**, where m and n are scalars. Find two distinct expressions for **OK** in terms of p,r,m and n. Hence find the values of m and n. (5mks) c) State the ratio PK:KS	
31	**2004 Q 4 P1** Given that **OA** = 3**i** -2**j** + and **OB** = 4**i** + **j** – 3**k**. Find the distance between points A and B to 2 decimal places.	Working Space

32	**2004 Q 21 P1** a) If A, B and C are the points P and Q are p and q respectively is another point with position vector r = $^{3}/_{2}$**q** -$^{1}/_{2}$ **p**. Express in terms of p and q. i) **PR** ii) **RQ** hence show that P, Q and R are collinear. iii) Determine the ratio PQ: QR.	
33	**2005 Q 13 P1** Point T is the midpoint of a straight line AB. Given the position vectors of A and T are i-j + k and 2i+ 1 ½ k respectively, find the position vector of B in terms of i, j and k. (3 marks)	
34	**2005 Q 18 P1** The points P, Q, R and S have position vectors 2**p**, 3**p**, **r** and 3**r** respectively, relative to an origin O. A point T divides PS internally in the ratio 1:6 (a) Find, in the simplest form, the vectors **OT** and **QT** in terms **P** and **r** (4 marks) (b) (i) Show that the points Q, T, and R lie on a straight line (3 marks) (ii) Determine the ratio in which T divides QR (1 mark)	

35 **2006 Q 12 P1**

Two points P and Q have coordinates (-2 ,3) and (1,3) respectively. A translation map point P to P' (10, 10)

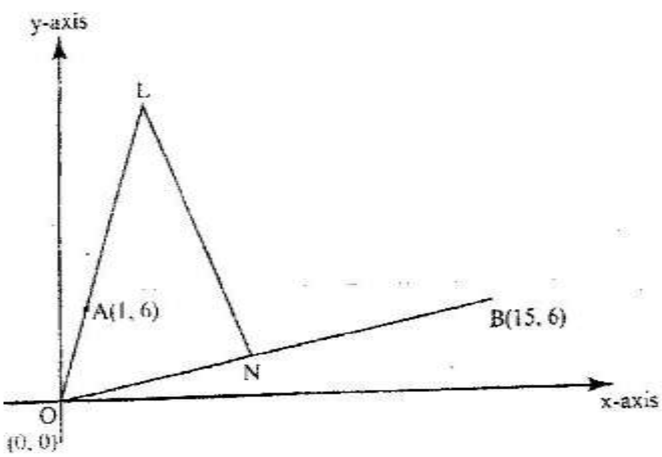
a) Find the coordinates of Q' the image of Q under the translation (1 mark)

(ii) The position vector of **P** and **Q** in (a) above are p and q respectively given that $m\mathbf{p} - n\mathbf{q} = \begin{pmatrix} -12 \\ 9 \end{pmatrix}$ (3 marks)

b) Find the value of *m* and *n*

36 **2006 Q 22 P1**

In the diagram below, the coordinates of points A and B are (1,6) and (15,6) respectively). Point N is on OB such that 3 ON = 2OB. Line OA is produced to L such that OL = 3 OA



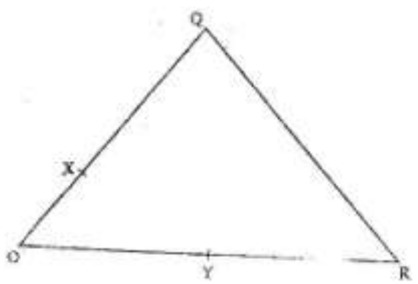
(a) Find vector LN (3 marks)

(b) Given that a point M is on LN such that LM: MN = 3: 4, find the coordinates of (2 marks)

(c) If line OM is produced to T such that OM: MT = 6:1

(i) Find the position vector of T (1 mark)

(ii) Show that points L, T and B are collinear (4 marks

37	**2006 Q 9 P2** Given that q **i** + $^1/_3$ **j** + $^2/_3$ **k** is a unit vector, find q (2 marks)	
38	**2007 Q 21 P1** In the figure below, **OQ** = q and **OR** = r. Point X divides OQ in the ratio 1: 2 and Y divides OR in the ratio 3: 4 lines XR and YQ intersect at E.  (a)Express in terms of q and r (i) **XR** (1 mark) (ii) **YQ** (1 mark) (b) If **XE** = m **XR** and **YE** = n **YQ**, express **OE** in terms of: (1 mark) (i) r, q and m (ii) r, q and n (1 mark) (c) Using the results in (b) above, find the values of m and n. (6 marks)	
39	**2007 Q 12 P2** Vector q has a magnitude of 7 and is parallel to vector p. Given that p= 3 **i** –**j** + 1 ½ **k**, express vector q in terms of i, j, and k. (2 marks)	Working Space

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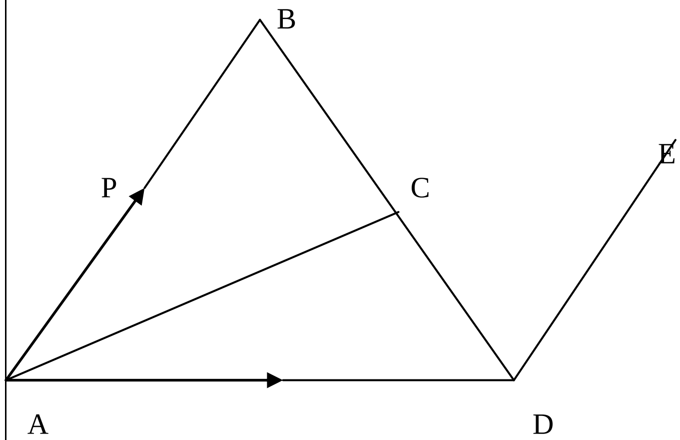
2008 Q 19 P2
In the figure below **AB=p, AD= q, DE= ½ AB and BC= $^2/_3$ BD**



a) Find in terms of p arid q the vectors: (1mk)

(i) **BD**; (1mk)
(ii) **BC**; (1mk)
(iii) **CD**; (1mk)
(iv) **AC**. (2mks)

b) Given that **AC**=*k***CE**, where *k* is a scalar, find
(i) The value of *k* (4mks)
(ii) The ratio in which C divides AE (1mk)

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2008 Q 4 P2

The position vectors of points A and B are $\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix}$ respectively.

A point P divides AB in AB it he ratio 2:3. Find the position Vector of point P. (3mks)

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2009 Q 20 P1
The position vectors of point A and B with respect to the O,are $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$ respectively
Point M is the midpoint of AB and N is the midpoint of OA.

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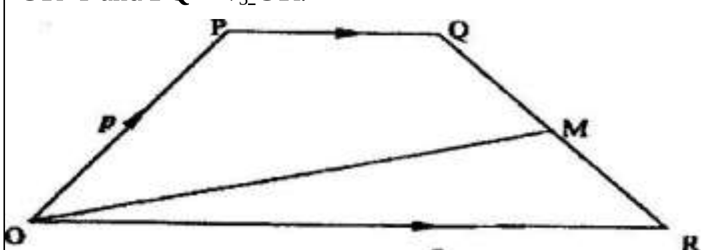
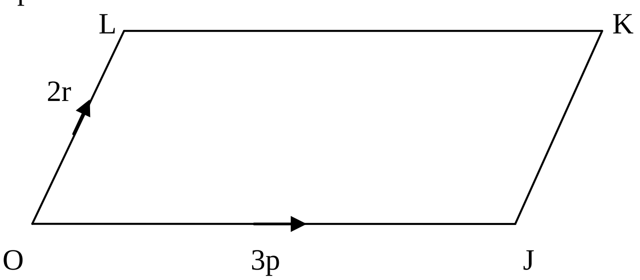
	(a) Find: i) The coordinates of N and M (3 mks) ii) The magnitude of NM (3 mks) (b) Express vector **NM** in term of **OB**. (c) Point P maps onto P by a translation $\begin{pmatrix}-5\\8\end{pmatrix}$ Given that **OP**=**OM**+2**MN**, find the coordinates of P´	
43	**2009 Q 6 P2** Vector **OA** =$\begin{pmatrix}2\\1\end{pmatrix}$ and **OB**= $\begin{pmatrix}6\\-3\end{pmatrix}$ Point C is on OB such CB=2OC and point D is on AB such that AD=3DB. Express **CD** as a column vector. (3 mks)	
44	**2010 Q 7 P1** In the figure below, OPQR is a trapezium in which PQ is parallel to OR and M is the mid-point of QR and **OP=p, OR=r** and **PQ** = $^1/_3$ **OR**.  Find **OM** in terms of **p** and **r.** (3 mks)	Working Space

45 **2010 Q 18 P2**

In the figure below OJKL is a parallelogram in which OJ = 3p and OL = 2r



a) If A is a point on LK such that LA = ½ AK and point B divide the line JK externally in the ratio 3:1, express **OB** and **AJ** in terms of **p** and **r.** (2 marks)

b) Line OB interests AJ at X such that **OX** = m**OB** and **AX** = n**AJ.**

i) Express **OX** in terms of **p, r** and m. (1 mark)

ii) Express **OX** in terms of **p, r** and n (1 mark)

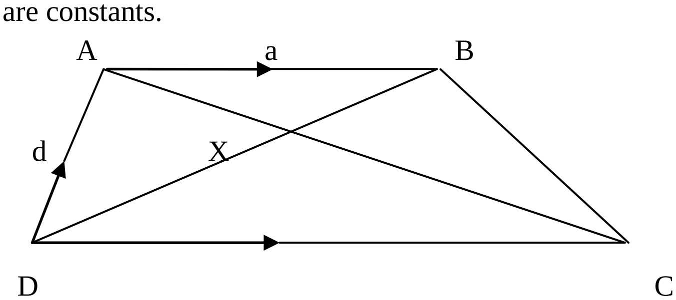
iii) Determine the value of *m* and *n* and hence the ratio in which point x divides line AJ. (6 marks)

46 **2011 Q 13 P2**

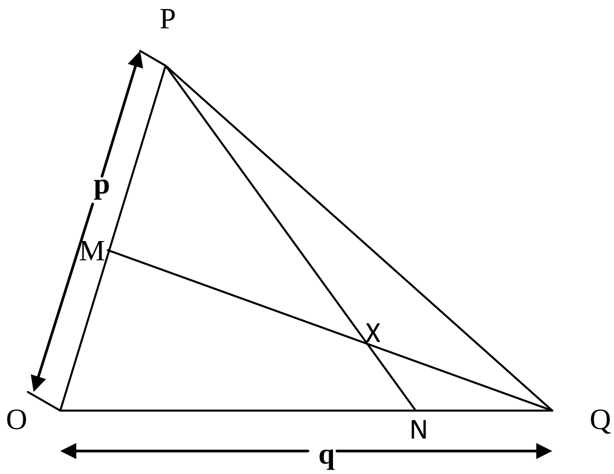
Vector **OP**=6**i** + **j** and **OQ**+ -2**i** +5**j**. A point N divides **PQ** internally in the ratio 3:1. Find **PN** in terms of i and j. (3 mks)

47 **2011 Q 23 P1**

In the figure below,ABCD is a trapezium. AB is parallel to DC,diagonals AC and DB intersect at X and DC=2AB. **AB= a, DA=d, AX=**k **AC** and **DX**= h**DB** where h and k are constants.



Working Space

	a) Find in terms of a and d i) BC (2 mks) ii) AX (2 mks) iii) DX (1 mks)	
48	**2012 Q4 P2** Given that $\boldsymbol{P}=2\boldsymbol{i}-3\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{Q}=3\boldsymbol{i}-4\boldsymbol{j}-3\boldsymbol{k}$ and $\boldsymbol{R}=3\boldsymbol{P}+2\boldsymbol{Q}$, find the magnitude of $\boldsymbol{R}$ to 2 significant figures. (3 marks)	
49	**2012 Q19 P2** In triangle OPQ below, **OP** = **p**, **OQ**=**q**. Point M lies on **OP** such that **OM**: **MP**=2:3 and point N lies on **OQ** such that **ON**: **NQ**=5:1. Line PN intersects line MQ at X.  (a) Express in terms of **p** and **q**: (i) **PN;** (1 mark) ii) **QM;** (1 mark) (b) Given that **PX**= k**PN** and **QX**=r**QM,** where k and r are scalars: (i) Write two different expressions for OX in terms of **p**,**q**,k and r; (2 marks) (ii) Find the values of k and r; (4 marks) (iii) Determine the ratio in which X divides line MQ. (2 marks)	
